On the relation between the surface area in contact and the coefficient of static friction

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Tiivistelmä

Yleinen käsitys ja fysiikan teoria ovat ristiriidassa keskenään kitkan pinta-alariippuvuuden suhteen: intuitiivisesti ajatellaan, että kosketuksessa olevan pinta-alan suuruus vaikuttaa kitkaan, kun taas fysiikassa tehdään usein olettama, että kitkakerroin ei riipu pinta-alasta. Tässä työssä tutkitaan kyseistä riippuvuussuhdetta.

Työ on rajattu staattiseen kitkaan ja ainoastaan pinta-alan vaikutukseen siihen, pintojen välinen paine pidetään samana. Staattinen kitkakerroin mitattiin kokeellisesti kolmelle erikokoiselle palalle neljällä eri materiaali yhdistelmällä käyttäen kaltevaa tasoa. Näin pyritään myös löytämään materiaalin vaikutus tutkittavaan riippuvuussuhteeseen, toisin sanoen millä tavoin suhde eroaa eri materiaalien välillä ja miksi. Tutkimuskysymys on: Miten kosketuksessa olevan pinta-alan suuruuden muuttaminen vaikuttaa staattiseen kitkakertoimeen kun pintojen välinen paine pidetään vakiona, ja onko vaikutus samanlainen eri materiaaleilla?

Materiaalien vaikutusta analysoidaan teoreettisesti atomi-tasolla. Tutkielmassa tultiin siihen lopputulokseen, että kitkan pinta-alariippuvuuteen liittyvät merkilliset ominaisuudet voidaan parhaiten selittää juuri atomi-tason analyysillä. Kokeista saaduista tuloksista päätellään, että staattinen kitkakerroin todellakin riippuu kosketuksessa olevan pinta-alan suuruudesta.

Mittalaitteiston melko suuren epätarkkuuden vuoksi lopputuloksiin kertyy niin paljon epätarkkuutta, että tutkittavalle riippuvuussuhteelle ei voida määritellä matemaattista kaavaa. Kokeesta saatiin kuitenkin merkittävä tulos siitä, kuinka pidon suuruus vaikuttaa kyseiseen riippuvuussuhteeseen, eli mitä parempi pito materiaalissa on, sitä vähemmän vaikutusta kosketuksessa olevan pinta-alan suuruudella on staattiseen kitkakertoimeen.

Abstract

The common conception and physical theory contradict on the surface area dependency of friction: intuition lets us think that friction is affected by the surface area in contact, whereas in Physics we often make the assumption that the coefficient of friction is independent on the surface area. This relation is investigated in this essay.

The scope of the investigation is limited to static friction and to investigate the effect of pure surface area, the surface pressure is held constant. An experiment was carried out to measure the coefficient of static friction for three different-sized pieces of four different material combinations. Thereby we also tried to find out the role of the material in this relation, in other words how the relation is different for different materials and why. The research question is then the following: How does altering the surface area in contact while holding the surface pressure constant affect the coefficient of static friction, and are the trends similar for different materials?

We tackle the part of the research question on the materials with atomic-scale analysis of friction. This is where the cause for non-intuitive frictional properties is suggested to lie at the end of this essay. From the results obtained in the experiment, we draw the conclusion that the coefficient of static friction is indeed dependent on the surface area in contact.

Because of the rather great imprecision of the experimental equipment used and the consequential uncertainty in the results, no mathematical relation can be confirmed. However, the experiment yields valuable results on the effect of the level of grip on the relation being investigated, namely that the better grip the material has, the less the coefficient of static friction is affected by the surface area in contact.

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Introduction

Friction is relevant to almost every aspect of our everyday lives and thus it is mostly recognised as something very intuitive. Basic techniques for obtaining a better grip when needed are generally known. The most common one is to simply employ a material that is known to have a better grip, in other words a greater coefficient of friction. This is where intuition plays a central role. However, if we wished to obtain a better grip using the same material, it becomes more complicated. To try to bring about a change in the frictional coefficient of a material we could, for example, apply the same normal force on a smaller area covered with the material or increase the surface area and normal force in the same proportion. In the former case, the pressure on the material would increase, whereas in the latter case it would stay the same.

There is a contradiction between physical theory and the general view of the surface area dependency of friction. There is a common conception that changing the surface area in contact affects the grip of a material, while in physics calculations, the assumption that friction is independent of the surface area in contact is usually made. In work [1] it says: “For given surfaces, experiment shows that the friction force is approximately proportional to the *normal force* between the two surfaces --. The force of friction between hard surfaces in many cases depends very little on the total surface area of contact”. This suggests that there actually is some surface area dependency and also that this may vary between materials. This is what aroused my interest toward the topic, and I decided to test this experimentally.

It is quite often in everyday life that friction is said to arise from surface roughness. However, a contradiction arises when then trying to explain why a very smooth looking surface, such as some rubber surfaces, has a very good grip. Here is another difference between a general conception and physical theory. In Physics, it is today generally accepted that frictional force is not a fundamental force, akin to normal pushes and pulls but is made up of electromagnetic forces between the atoms on a surface. Hence, the “roughness” causing friction, is actually molecular-scale roughness, if we can talk about roughness at all in this case. This means that friction is dependent on molecular-scale properties and this theory explains very nicely, why the naked eye cannot necessarily tell if a material has a great coefficient of friction, and can possibly account for other non-intuitive properties of friction that arise in this investigation.

Since there is a variety of factors that possibly affect the coefficient of friction, and there are also different kinds of friction, the scope of this investigation was further narrowed. I chose to investigate the effects on static friction only, since this is probably the simplest type of friction, hence involving the least number of factors that could cause a deviation from a trend between the area of contact and the frictional coefficient and decrease the accuracy of the results. An apparent factor involved is pressure, and in this investigation it is held constant to see the effect of the pure change in the surface area in contact. Hence the research question is: How does altering the surface area in contact while holding the surface pressure constant affect the coefficient of static friction, and are the trends similar for different materials?

It is found in this essay that altering the surface area changes the coefficient of static friction. Partly due to the imprecision of the experimental equipment no mathematical relation can be confirmed, although there are certain trends present. The trends are found to be dissimilar for different materials, and the grip of the material is suggested to be the property that defines the dependency of the coefficient of static friction on the surface area in contact, because in the results obtained, the better grip the material has the less difference there is between the frictional coefficients measured for different surface areas.

The Experiment

Experimental setup

List of apparatus:

- hardboard as the inclined plane
- lift
- two piles of stiff material as platform for the lift
- measuring stick
- test materials:
  - large pieces of cardboard and rubber mat
  - 5cm*5cm, 10cm*10cm and 20cm*20cm pieces of cardboard and rubber mat

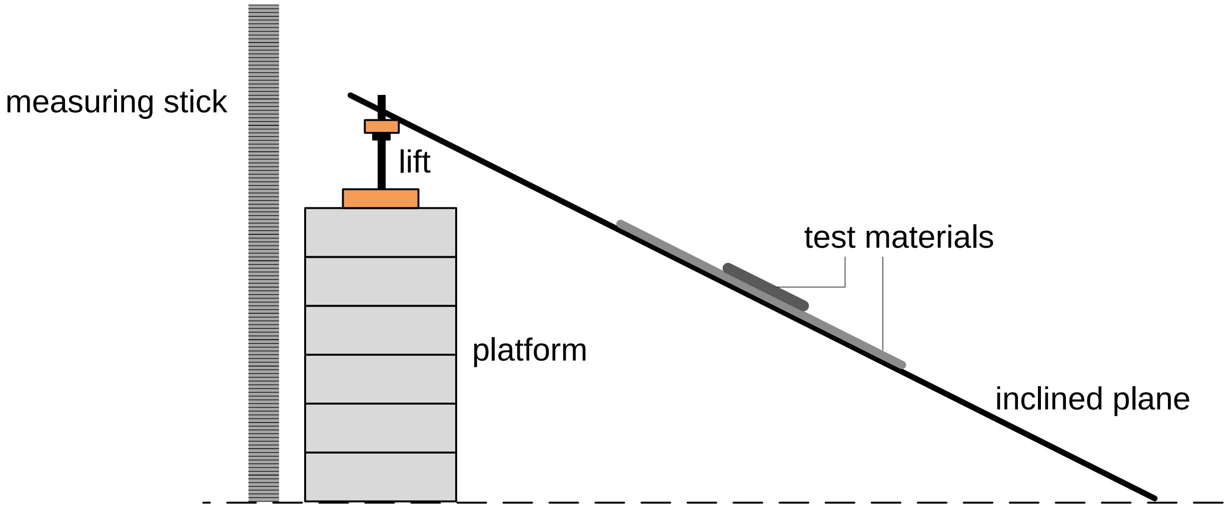

Figure 1: Experimental setup

It is vital to check that the base on which the setup is constructed is horizontal, in other words perpendicular to gravity, and that the measuring stick and the lower end of the inclined plane are both at the same level. This can be done by first using a strip of rigid material that is known to have been cut straight to check that the base is not curved and then using a spirit level to check that it is horizontal.

For a photograph of the experimental setup, see Figure 4 in the Appendix.

Procedure

An inclined plane is used for measuring the coefficients of static friction. The apparatus is set up as shown in Figure 1. A large piece of the material, which is used as the bottom material, is attached on the hardboard which is used as the inclined plane. On this material, we set the test piece of material of certain area, and then slowly increase the angle of inclination until the piece of material starts sliding down. The height of the end of the plane is then recorded, and from this and the length of the plane, the angle of inclination can be obtained applying simple geometry, according to Figure 3. Figure 2 represents the situation just before the piece of material starts sliding down, in other words just before gravity overcomes static friction, so that angle α represents the critical angle, which is the maximum angle of inclination at which static friction is enough to keep the test piece in place.

As the idea is to measure the coefficient of static friction for different-sized pieces of the same material, I decided to test three different sizes: 5cm*5cm, 10cm*10cm and 20cm*20cm. Now, since the areas are easily compared, the second one being four times the first one, the third one being four times the second one and 16 times the first one, we could possibly find some proportionality relation between the areas and coefficients of static friction, if there are differences between the coefficients.

The measurement is repeated five times for each piece of material. After each trial the inclined plane is lowered enough for the test material to stay in place on it, and then the inclination is slowly and steadily increased again until the piece of material starts sliding. For each material, the height of the platform of the lift is adjusted so that the range of the lift will be enough to reach the critical angle. The lift used consisted simply of two bolts attached on the ends of a plank, and to the bolts were screwed nuts on which a strip of wood rested. The strip of wood could then be lifted rather steadily when screwing the nuts simultaneously.

To see if the relation between the surface area and static friction is different for different materials, I used rubber mat, whose other side is corrugated, and the other smooth, and cardboard as the test materials. I chose these two materials on the basis that rubber is known to have a rather high coefficient of friction, and cardboard to be more slippery. If there are differences, this could also yield interesting results about the effect of how good grip the material has on the relation between surface area in contact and the coefficient of static friction.

I measured the coefficient of static friction for the rubber mat in three different situations: the smooth side against the smooth side, the smooth side against the corrugated side, and the corrugated side against the corrugated side. For the cardboard, the sides were also different from each other but only one side seemed homogeneous enough for the material to be similar at each point of the three pieces, and so only that side was used.

Originally I also wanted to test with a third material that would have been very slippery, to have a large scale of coefficients of static friction. This would have been a more effective way of tackling the part of the research question on different materials, more accurately materials that defer in how good grip they have. The material I had chosen for this was Teflon® tape and the counter-material a hard piece of plastic lubricated with silicone spray and polished. However, this combination proved so slippery that the critical angle was only a few degrees, and the inclination had to be zero when setting the test material on the plane to get it stay in place. Also, even a slight shaking of the plane when lifting the end would make the material start sliding down before the critical angle was reached. Consequently, the deviation of the few results obtained for this material was so great that the uncertainty seemed too great for the results to be of value when comparing to the results for the two other materials. That the critical angle was small also increased the percent uncertainty to a rather great value as compared to the other results.

Calculation of the coefficient of static friction from the experimental data

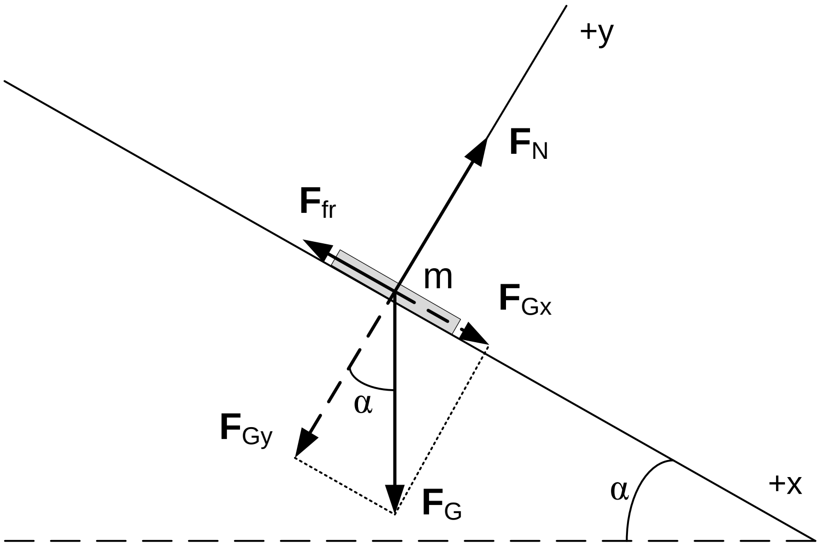

Figure 2: Free-body diagram for the piece of material on the inclined plane

In Figure 2 the forces are as follows:

F$_N$: Normal force that the inclined plane exerts on the piece of material

F$_G$: Gravitational force on the mass m of the piece of material

F$_{Gy}$: Component of **F**$_G$ along the y-axis

F$_{Gx}$: Component of **F**$_G$ along the x-axis

F$_{fr}$: Frictional force on the piece of material

We derive a formula for the coefficient of static friction μ_s:

$$F_G = mg$$
$$F_N = F_{Gy}$$
$$F_{Gy} = F_G \cos\alpha = mg\cos\alpha$$
$$F_N = mg\cos\alpha$$
$$F_{fr} = \mu_s F_N = \mu_s mg \cos\alpha$$
$$F_{fr} = F_{Gx} = mg\sin\alpha$$
$$\mu_s mg \cos\alpha = mg\sin\alpha$$
$$\mu_s = \frac{\sin\alpha}{\cos\alpha} = \tan\alpha$$

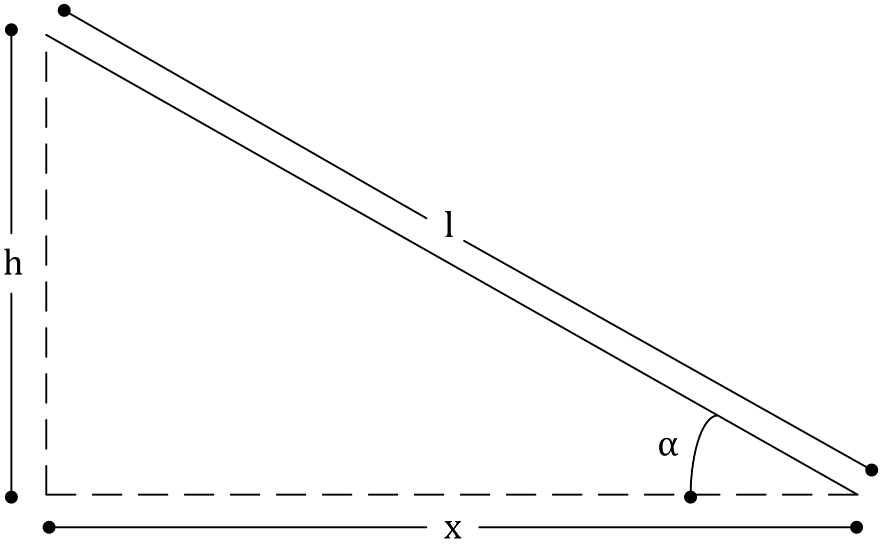

Figure 3: Geometry of the inclined plane

In Figure 3:

h: Height of the end of the plane

l: Length of the plane

x: Horizontal distance under the plane

We derive a formula for the tangent of the critical angle:

$$h^2 + x^2 = l^2$$
$$x = \sqrt{l^2 - h^2}$$
$$\tan\alpha = \frac{h}{x} = \frac{h}{\sqrt{l^2 - h^2}} = \frac{1}{\sqrt{\frac{l^2}{h^2} - 1}}$$

Now, since the tangent of the critical angle equals the coefficient of static friction μ_S, we have:

$$\mu_s = \tan\alpha = \frac{1}{\sqrt{\frac{l^2}{h^2} - 1}}$$

So, by first measuring the length of the hardboard used as the inclined plane l, and then the height of the end of the plane as the test piece of material starts sliding, we can obtain a value for the coefficient of static friction using this formula.

Variables

Independent:

- Size of the piece of material, in other words the surface area in contact

Dependent:

- Coefficient of static friction

Controlled:

- Pressure
- Temperatures

The size of the piece of material is the independent variable that is what we alter, and the coefficient of static friction is classified as the dependent variable, as we assume some dependency to exist.

I wanted the pressure to be the same for each of the three pieces of the same material, to investigate the effect of pure surface area, since if wooden blocks, say, would be used, and turned from a side of greater surface area to a side of smaller surface area, which is a typical test carried out when investigating this phenomenon, the difference in pressure could account for a difference in the frictional coefficient. The overall pressure is constant since we use pieces of different area but of the same thickness, which maintains the force per unit area quantity.

The experiment is carried out in room temperature and it is assumed to be constant enough throughout the experiment for its effect to be neglected.

Uncertainties

The uncertainties related to the testing apparatus itself, measurement, and the materials have to be considered very carefully. First of all we consider the uncertainties related to the measurement. The measuring was carried out using a basic measuring stick. We take the instrumental uncertainty to be ±0.1cm, the smallest unit on their measuring scale. When carefully measuring, not a great human error will be involved in the results, or at least the instrumental uncertainty will include this error. The total uncertainty for the measuring process will then be ±0.1cm. This uncertainty will be involved in measuring the height of the end of the inclined plane and the length of the plane, as well as in measuring the surface areas of the pieces of material used in the tests.

Another, and probably the most important factor causing uncertainty, is the non-homogeneity of the materials, i.e. that the materials are not similar at each point on the surface, and on all three different pieces. The cardboard was taken from only one cardboard box, and there were no creases, and the material looked very similar to the eye on all three resulting pieces. Also the rubber mat was cut from a large roll of very similar-looking material. However, frictional forces are today accounted for with molecular-scale properties. Consequently the naked eye cannot tell whether the material is perfectly homogenous, when it comes to frictional properties. It could also be noticed during the experiment, that the base material was not exactly the same at each point, since with the same angle of inclination, at slightly different locations, with the cardboard especially, there was a clear difference in how well the test piece of material would stay in place.

The uncertainty the testing apparatus itself causes may appear when increasing the inclination of the plane, if the lift provides slightly jerky movement. Then, the static friction between the test materials may turn to kinetic friction as a consequence of the vibration of the supporting plane, before the maximum inclination is reached, at which the material would just start to move by the effect of pure gravitation. I tried to minimise this uncertainty by designing the lift so that the upward movement would be as smooth as possible.

The two latter sources of uncertainty are such that they also affect the precision of the test, i.e. the repeatability of the test, or how close to the same value you can get in a new trial with the same setup.

Data collection

The length of the hardboard used as the inclined plane l was measured to be:

$$l = (80.0\ \pm 0.1)cm$$

The surface areas are measured by cutting a square of certain side. In measuring the length of the side of the square x, the uncertainty $\pm\delta x$ is: ±0.1cm. The formula used in calculating the resulting uncertainty in the area A is explained in the Results section below.

For each data set, in other words the five values measured for the height of the end of the plane h for each size and material, we first calculate the mean value $\bar{h}$. This is the sum of the set of values divided by their number, which is here five. Standard deviation is a measure of spread of the values of a data set, and we employ it in calculating the uncertainty. Since we have such a small set of data, we use the sample standard deviation rather than the population standard deviation. This will produce slightly greater values for the standard deviation, and eventually for the uncertainty. The sample standard deviation of the height σ_h is calculated using the formula:

$$\sigma_h = \sqrt{\frac{\sum\left(h - \bar{h}\right)^2}{n - 1}}$$

where $\bar{h}$ is the mean height and n is the number of items in the data set.

The absolute uncertainty of the height $\pm\delta h$ is then calculated using the formula:

$$\pm\delta h = \pm\frac{\sigma_h}{\sqrt{n}}$$

Length of the side of the square **x / cm**		**5.0 ± 0.1**	**10.0 ± 0.1**	**20.0 ± 0.1**
Surface area **A / cm^2**		**25 ± 1**	**100 ± 2**	**400 ± 4**
Height **h / cm**	**#1**	46.9	46.5	54.9
	#2	52.0	47.3	55.1
	#3	55.1	47.9	55.5
	#4	54.8	47.9	55.7
	#5	54.7	47.6	55.4
Mean value of h **$\bar{h}$ / cm**		52.70	47.44	55.32
Standard deviation of h **σ_h**		3.475	0.5814	0.3194
Absolute uncertainty in h **$\pm\delta h$**		1.554	0.2600	0.1428
Relative uncertainty in h **$\pm\frac{\delta h}{\bar{h}}$ / %**		2.95 %	0.548 %	0.258 %

Table 1: Data for corrugated rubber on corrugated rubber

Length of the side of the square **x / cm**		**5.0 ± 0.1**	**10.0 ± 0.1**	**20.0 ± 0.1**
Surface area **A / cm^2**		**25 ± 1**	**100 ± 2**	**400 ± 4**
Height **h / cm**	**#1**	57.1	55.3	59.9
	#2	56.5	54.7	59.0
	#3	56.4	54.7	58.8
	#4	55.6	54.3	58.6
	#5	56.1	56.3	58.6
Mean value of h **$\bar{h}$ / cm**		56.34	55.06	58.98
Standard deviation of h **σ_h**		0.550	0.7797	0.5404

Absolute uncertainty in *h* **$\pm\delta h$**	0.2462	0.3487	0.2417
Relative uncertainty in *h* $\pm\frac{\delta h}{\bar{h}}$ **/ %**	0.437 %	0.633 %	0.410 %

Table 2: Data for smooth rubber on corrugated rubber

Length of the side of the square ***x* / cm**		**5.0 ± 0.1**	**10.0 ± 0.1**	**20.0 ± 0.1**
Surface area **A / cm^2**		**25 ± 1**	**100 ± 2**	**400 ± 4**
Height ***h* / cm**	**#1**	65.5	64.6	64.1
	#2	65.6	64.0	64.0
	#3	64.6	63.8	63.7
	#4	64.6	65.3	63.8
	#5	64.5	64.1	63.9
Mean value of *h* **$\bar{h}$ / cm**		64.96	64.36	63.90
Standard deviation of *h* **σ_h**		0.5413	0.6025	0.1581
Absolute uncertainty in *h* **$\pm\delta h$**		0.2421	0.2694	0.07071
Relative uncertainty in *h* $\pm\frac{\delta h}{\bar{h}}$ **/ %**		0.373 %	0.419 %	0.111 %

Table 3: Data for smooth rubber on smooth rubber

Length of the side of the square ***x* / cm**		**5.0 ± 0.1**	**10.0 ± 0.1**	**20.0 ± 0.1**
Surface area **A / cm^2**		**25 ± 1**	**100 ± 2**	**400 ± 4**
Height ***h* / cm**	**#1**	36.1	43.9	32.9
	#2	42.6	43.7	34.7
	#3	51.6	41.1	33.8
	#4	45.5	40.4	33.6
	#5	47.5	46.6	33.4
Mean value of *h* **$\bar{h}$ / cm**		44.66	43.14	33.68
Standard deviation of *h* **σ_h**		5.798	2.476	0.6611

Absolute uncertainty in h $\pm\delta h$	2.593	1.108	0.2956
Relative uncertainty in h $\pm\frac{\delta h}{\bar{h}}$ / %	5.81 %	2.57 %	0.878 %

Table 4: Data for cardboard on cardboard

Results

We define the coefficient of static friction μ_S as a function of the height h and length l, according to the formula derived above:

$$\mu_S(h, l) = \frac{1}{\sqrt{\frac{l^2}{h^2} - 1}}$$

To calculate the absolute uncertainty in the final results, in other words the coefficients of static friction, $\pm\delta\mu_S$, based on work [3] we use the following formula:

$$\delta\mu_S = \sqrt{\left[\frac{\partial\mu_s}{\partial h}\cdot\delta h\right]^2 + \left[\frac{\partial\mu_s}{\partial l}\cdot\delta l\right]^2}$$

where $\frac{\partial\mu_s}{\partial h}$ is the partial differential of the function μ_s with respect to variable h

$\frac{\partial\mu_s}{\partial l}$ is the partial differential of the function μ_s with respect to variable l

δh is the absolute uncertainty in variable h

δl is the absolute uncertainty in variable l

Evaluating the partial differentials and simplifying, we obtain:

$$\delta\mu_s = \sqrt{\left[\frac{1}{l\left(\sqrt{1 - \frac{h^2}{l^2}}\right)^3}\cdot\delta h\right]^2 + \left[\frac{l}{h^2\left(\sqrt{\frac{l^2}{h^2} - 1}\right)^3}\cdot\delta l\right]^2}$$

In this experiment, the length l and the related uncertainty δl is the same for each measured value of the height h. The above formula is used to calculate the uncertainty in the coefficient of static friction, substituting the mean height $\bar{h}$ for h, and using the related uncertainty δh in each case.

We define the surface area of the test piece of material A as a function of the length of the side of the square x as follows:

$$A(x) = x^2$$

Based on work [3] we use the following formula:

$$\delta A = \frac{dA}{dx} \cdot \delta x$$

where $\frac{dA}{dx}$ is the derivative of the function A with respect to x

δx is the absolute uncertainty in x

Evaluating the derivative:

$$\delta A = 2x \cdot \delta x$$

This is the formula, which is used to calculate the absolute uncertainty in the surface area A.

Length of the side of the square x / cm	**5.0 ± 0.1**	**10.0 ± 0.1**	**20.0 ± 0.1**
Surface area A / cm^2	**25 ± 1**	**100 ± 2**	**400 ± 4**
Coefficient of static friction μ_S	0.8756	0.7365	0.9573
Absolute uncertainty in μ_S $\pm\delta\mu_S$	0.04565	0.006385	0.005262
Relative uncertainty in μ_S $\pm(\delta\mu_S/\mu_S)$ / %	5.21 %	0.867 %	0.550 %
$\mu_S \pm \delta\mu_S$	0.876 ± 0.046	0.737 ± 0.007	0.957 ± 0.006

Table 5: Results for corrugated rubber on corrugated rubber

Length of the side of the square x **/ cm**	**5.0 ± 0.1**	**10.0 ± 0.1**	**20.0 ± 0.1**
Surface area A **/ cm^2**	**25 ± 1**	**100 ± 2**	**400 ± 4**
Coefficient of static friction μ_S	0.9920	0.9487	1.091
Absolute uncertainty in μ_S $\pm\delta\mu_S$	0.008944	0.01164	0.01024
Relative uncertainty in μ_S $\pm(\delta\mu_S/\mu_S)$ **/ %**	0.902 %	1.23 %	0.938 %
$\mu_S \pm \delta\mu_S$	0.992 ± 0.009	0.949 ± 0.012	1.09 ± 0.02

Table 6: Results for smooth rubber on corrugated rubber

Length of the side of the square x **/ cm**	**5.0 ± 0.1**	**10.0 ± 0.1**	**20.0 ± 0.1**
Surface area A **/ cm^2**	**25 ± 1**	**100 ± 2**	**400 ± 4**
Coefficient of static friction μ_S	1.391	1.354	1.328
Absolute uncertainty in μ_S $\pm\delta\mu_S$	0.01605	0.01678	0.006122
Relative uncertainty in μ_S $\pm(\delta\mu_S/\mu_S)$ **/ %**	1.15 %	1.24 %	0.461 %
$\mu_S \pm \delta\mu_S$	1.39 ± 0.02	1.35 ± 0.02	1.33 ± 0.01

Table 7: Results for smooth rubber on smooth rubber

Length of the side of the square *x* / cm	**5.0 ± 0.1**	**10.0 ± 0.1**	**20.0 ± 0.1**
Surface area *A* / cm^2	**25 ± 1**	**100 ± 2**	**400 ± 4**
Coefficient of static friction μ_S	0.6729	0.6403	0.4641
Absolute uncertainty in μ_S $\pm\delta\mu_S$	0.05676	0.02321	0.005002
Relative uncertainty in μ_S $\pm(\delta\mu_S/\mu_S)$ / %	8.44 %	3.62 %	1.08 %
$\mu_S \pm \delta\mu_S$	0.673 ± 0.057	0.640 ± 0.024	0.464 ± 0.006

Table 8: Results for cardboard on cardboard

Analysis of results and Discussion

Looking at the results, it is obvious that the coefficient of static friction is not completely independent on the surface area in contact. We can arrive at this conclusion, because for each of the four different material combinations tested, a different surface area of contact has a different coefficient of static friction.

No direct mathematical relation between the surface area in contact and the coefficient of static friction can however be found, since the results for different materials contradict. According to the results for the experiments with corrugated rubber on corrugated rubber and smooth rubber on corrugated rubber, the 100 cm^2 piece has the smallest, the 25 cm^2 piece the second smallest and the 400 cm^2 the greatest coefficient of static friction. On the other hand, the results for smooth rubber on smooth rubber and cardboard on cardboard suggest that the largest surface area has the smallest coefficient of static friction and vice versa.

Overall, the results of this experiment seem to be more of qualitative than quantitative value, providing us with indications of trends rather than actual relations. We can thereby answer the part of the research question about trends between the surface area in contact and the coefficient of static friction for different materials, that the trends are dissimilar for different materials.

For Tables 5 and 6, plotting the results would not help in forming a mathematical relation, since the surface areas and corresponding values for the coefficient of static friction are neither directly nor inversely proportional. For Tables 7 and 8, the larger the surface area the smaller the coefficient of static friction. However, it is still obvious that the two quantities are not inversely proportional, as when surface area increases by a factor of four, the frictional coefficient only decreases to a much greater fraction than the reciprocal of this, in each case.

Although we cannot form a mathematical relation, we can analyse qualitatively how altering the surface area affects the coefficient of static friction differently for different materials. To do this, we calculate the mean and standard deviation of the three values for the frictional coefficient for each combination of materials:

Mean value of μ_S	0.856
Standard deviation of μ_S	0.112

Table 9: Corrugated rubber on corrugated rubber

Mean value of μ_S	1.01
Standard deviation of μ_S	0.0731

Table 10: Smooth rubber on corrugated rubber

Mean value of μ_S	1.36
Standard deviation of μ_S	0.0320

Table 11: Smooth rubber on smooth rubber

Mean value of μ_S	0.592
Standard deviation of μ_S	0.112

Table 12: Cardboard on cardboard

In these results, we have an interesting trend: The greatest coefficient of friction is that for the smooth rubber on smooth rubber combination, which however has the smallest standard deviation between the results for different areas. The second greatest frictional coefficient is that for the smooth rubber on corrugated rubber combination, which has the second smallest standard deviation. The second smallest and the smallest frictional coefficients are those of the corrugated rubber on corrugated rubber and cardboard on cardboard combinations respectively. These two have the same standard deviation, which has the greatest value.

The standard deviation here describes the extent to which surface area affects the frictional coefficient, in other words the greater the standard deviation is, the more the frictional coefficients of different areas of the material differ, that is the more effect surface area has on friction. As the trend here is, that the greater the coefficient, the less the standard deviation, we draw a conclusion that the better grip the material has, the less the surface area in contact affects the coefficient of static friction.

This raises the question: What properties of the material exactly define the extent to which surface area affects the coefficient of static friction and why? The simple answer given by our results is that it is the grip of the material but why?

According to work [2] and source [4], it is only a small fraction of the total surface area, where atomically close contact occurs. This is also where friction is given rise to, since it occurs as a result of the interaction between the atoms of the two materials in contact. For a material with a better grip, the interaction between the atoms is stronger but it is also likely that there are more points of contact per unit area. The fewer points of atomically close contact per unit area, the more likely a change in the surface area and hence also a change in the amount of points of contact is to affect friction. This conclusion is based on the random occurrence of the points of atomically close contact, in other words when there is overall fewer points of contact in the material, the additional surface area can contain a relatively great or small amount of new points of contact, whereas for a material with a great amount of points of contact overall, the change in the number of them is likely to be relatively less. This can be used to explain why the material with the best grip had the smallest surface area dependency of friction.

Another interesting property of materials when it comes to friction is the extent to which they are adhesive. Work [2] suggests that “friction and adhesion have some interrelations” and source [4] that “When the surfaces are adhesive, [the assumption that friction is independent on surface area] becomes a very poor approximation (for example, Scotch tape resists sliding even when there is no normal force, or a negative normal force). In this case, the frictional force may depend strongly on the area of contact.” Hence adhesion is likely to be another property of material that defines the extent to which its frictional coefficient is dependent on surface area. In this experiment none of the materials used were adhesive.

Conclusion and Evaluation

The relation between the surface in contact and the coefficient of static friction is a complex one, since no consistent relation emerged from the results. We proved a dependency to exist, however. Also, the grip of the material was found to be one determining factor on the extent to which surface area affects the coefficient of static friction.

However, taking into account the imprecision of the experimental equipment and the consequential uncertainty in the results, we cannot take the result about the effect of grip as positive evidence. What the results confirm, however, after careful consideration of the uncertainty, is that the coefficient of static friction is not independent of surface area, although it is often claimed to be.

The idea in this experiment was to keep the surface pressure constant, to allow the investigation of the effect of pure surface area. Although the overall surface pressure is the same for all the different-sized pieces of material, local pressures may vary, due to the occurrence of points of atomically close contact at different locations. This may one reason for the dependency of the coefficient of static friction on surface area. The cause of the dependency is hence likely to arise from the atomic level but the actual reason remains as an unresolved question.

The inclination can also be said to upset the overall pressure a little, so that highest local pressures would be found at the lower end of the piece of material. Devising an experiment to be carried out on horizontal plane would eliminate the possible effect of this.

Because the equipment used in this experiment has a rather low precision, we cannot be confident about obtaining results that are very close, when repeating the measurement. This is why repeating the experiment 30 times, say, is unlikely to result in a substantial decrease in the uncertainty of the results, as compared to only taking five results as was done. Hence, it is the precision of the experimental equipment that should be improved before repeating the experiment. After that considerably more results should be taken to decrease the uncertainty.

The problem with this investigation was the non-homogeneity of the materials at the atomic level. This problem persists until very high precision techniques are used to manufacture the test materials.

Adhesion was suggested as another property determining the dependency we were investigating, and overall to have interconnections with friction. The reliability of source [4] can be questioned, and therefore the implication should be verified by other means. The way adhesion affects the dependency of the coefficient of static friction on surface area, and the whole relation between adhesion and friction remain as unresolved questions.

Bibliography

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[4] Friction
http://en.wikipedia.org/wiki/Friction

Appendix

Figure 4: Photograph of the experimental setup